

# 高校数学「展開と因数分解・解答」

次の式を展開せよ。(厳格に公式を使おう)

①  $(2x + 3)(4x + 5) = 8x^2 + 22x + 15$

②  $(3x + 2)(2x + 7) = 6x^2 + 25x + 14$

③  $(5x - 3y)(4x + y) = 20x^2 - 7xy - 3y^2$

④  $(6a - b)(3a - 4b) = 18a^2 - 27ab + 4b^2$

⑤  $(5x + 4)^2 = 25x^2 + 40x + 16$

⑥  $(3x + 2)^2 = 9x^2 + 12x + 4$

⑦  $(4x - 3y)^2 = 16x^2 - 24xy + 9y^2$

⑧  $(7a - 5b)^2 = 49a^2 - 70ab + 25b^2$

⑨  $(2x + 3)(2x - 3) = 4x^2 - 9$

⑩  $(8x + 5)(8x - 5) = 64x^2 - 25$

⑪  $\left(\frac{1}{2}x + 5y\right)\left(\frac{1}{2}x - 5y\right) = \frac{1}{4}x^2 - 25y^2$

⑫  $(10a - 7b)(10a + 7b) = 100a^2 - 49b^2$

⑬  $(x + 2y - 3)^2 = x^2 + 4y^2 + 9 + 4xy - 12y - 6x$

⑭  $(2x + y + 5)^2 = 4x^2 + y^2 + 25 + 4xy + 10y + 20x$

⑮  $(3a - 2b + 4c)^2 = 9a^2 + 4b^2 + 16c^2 - 12ab - 16bc + 24ac$

⑯  $(4x - 3y - 2z)^2 = 16x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 16xz$

⑰  $(2x + 7y)^3 = 8x^3 + 84x^2y + 294xy^2 + 343y^3$

⑱  $(3x - 2)^3 = 27x^3 - 54x^2 + 36x - 8$

⑲  $(4x + 3y)^3 = 64x^3 + 144x^2y + 108xy^2 + 27y^3$

⑳  $(5a - 2b)^3 = 125a^3 - 150a^2b + 60ab^2 - 8b^3$

㉑  $(3x - 10y)(9x^2 + 30xy + 100y^2) = 27x^3 - 1000y^3$

㉒  $(2x - 5)(4x^2 + 10x + 25) = 8x^3 - 125$

㉓  $(6a - b)(36a^2 + 6ab + b^2) = 216a^3 - b^3$

㉔  $(4x + 3y)(16x^2 - 12xy + 9y^2) = 64x^3 + 27y^3$

㉕  $(x^2 + 3x + 6)(x^2 - 5x + 6) = x^4 - 2x^3 - 3x^2 - 12x + 36$

㉖  $(a^2 - 6a + 5)(a^2 + 6a - 5) = a^4 - 36a^2 + 60a - 25$

㉗  $(a - b + c - d)(a - b - c + d) = a^2 - 2ab + b^2 - c^2 + 2cd - d^2$

㉘  $(x + y)(x - y)(x^2 + y^2)(x^4 + y^4)(x^8 + y^8) = x^{16} - y^{16}$

㉙  $(x - y)^2(x + y)^2(x^2 + y^2)^2 = x^8 - 2x^4y^4 + y^8$

㉚  $(a + b + c)^3 = a^3 + 3a^2b + 3ab^2 + b^3 + 3a^2c + 6abc + 3b^2c + 3ac^2 + 3bc^2 + c^3$

㉛  $(x + y)^2(x - y)^2(x^2 + xy + y^2)^2(x^2 - xy + y^2)^2 = x^{12} - 2x^6y^6 + y^{12}$

㉜  $(x^2 + x + 1)(x^2 - x + 1)(x^4 - x^2 + 1) = x^8 + x^4 + 1$

㉝  $(x - 1)(x - 3)(x - 5)(x - 7) = x^4 - 16x^3 + 86x^2 - 176x + 105$

㉞  $(x - y + 1)(-x + y + 1)(x + y - 1)(x + y + 1) = -x^4 + 2x^2y^2 - y^4 + 2x^2 + 2y^2 - 1$

㉟  $(a + b + c)^2 + 2(a - b + c)^2 - 3(a + b - c)^2 = -8ab + 12ac + 4bc$

㊱  $(a + b + c)^2 - (a + b - c)^2 - (a - b + c)^2 + (-a + b + c)^2 = 8bc$

㊲  $x^5$  の係数は 40。係数の和  $\boxed{\text{ア}} + \boxed{\text{イ}} + \boxed{\text{ウ}} + \boxed{\text{エ}} + \boxed{\text{オ}} + \boxed{\text{カ}} + \boxed{\text{キ}} + \boxed{\text{ク}} + \boxed{\text{ケ}}$  の値は 225。

次の式を因数分解せよ。

- ①  $2x^2 + 7x + 3 = (x + 3)(2x + 1)$       ②  $3x^2 + 8x - 3 = (x + 3)(3x - 1)$   
③  $6x^2 + 11x + 3 = (2x + 3)(3x + 1)$       ④  $2x^2 - 17x + 15 = (x - 1)(2x - 15)$   
⑤  $4x^2 - 4x - 15 = (2x - 5)(2x + 3)$       ⑥  $6x^2 - 17x + 12 = (2x - 3)(3x - 4)$   
⑦  $2x^2 + 15xy + 18y^2 = (2x + 3y)(x + 6y)$       ⑧  $4a^2 + 5ab - 6b^2 = (4a - 3b)(a + 2b)$   
⑨  $6y^2 - 7xy - 3x^2 = (2y - 3x)(3y + x)$       ⑩  $8p^2 - 22pq + 15q^2 = (4p - 5q)(2p - 3q)$   
⑪  $10x^2 - 7xy - 12y^2 = (2x - 3y)(5x + 4y)$       ⑫  $4a^2 + 2a - 12 = 2(a + 2)(2a - 3)$   
⑬  $12x^2 - 44xy - 16y^2 = 4(x - 4y)(3x + y)$       ⑭  $c^2(a - b) + 9(b - a) = (a - b)(c - 3)(c + 3)$   
⑮  $16x^2 - (3x - 2y)^2 = (7x - 2y)(x + 2y)$   
⑯  $(x + 3y)(x + 3y - 7) + 12 = (x + 3y - 4)(x + 3y - 3)$   
⑰  $2(a + b)^2 - a - b - 3 = (a + b + 1)(2a + 2b - 3)$   
⑱  $(2x - y)^2 - (x + 2y)^2 = (x - 3y)(3x + y)$       ⑲  $(a + b)^4 - (a - b)^4 = 8ab(a^2 + b^2)$   
⑳  $x^2 - y^2 + 4y - 4 = (x - y + 2)(x + y - 2)$       ㉑  $x^3 + x^2 + 2x + 2 = (x + 1)(x^2 + 2)$   
㉒  $2a^3 - 1 + a^2 - 2a = (2a + 1)(a - 1)(a + 1)$       ㉓  $4b - 12 - (b - 3)^3 = -(b - 1)(b - 3)(b - 5)$   
㉔  $xy^2 + y + z - xz^2 = (y + z)(xy - xz + 1)$   
㉕  $a^3 + a^2b - ac^2 - bc^2 = (a + b)(a - c)(a + c)$   
㉖  $ab^2 - 4ab + 3a - b + 3 = (b - 3)(ab - a - 1)$   
㉗  $x^3 + (a - 2)x^2 - (2a + 3)x - 3a = (x + 1)(x - 3)(a + x)$   
㉘  $x^2 + 2xy + y^2 + 3x + 3y + 2 = (x + y + 2)(x + y + 1)$   
㉙  $2x^2 - xy - y^2 + 5x + y + 2 = (x - y + 2)(2x + y + 1)$   
㉚  $6x^2 - 7xy - 3y^2 - 11y - 6 = (-3x - y - 3)(-2x + 3y + 2)$   
㉛  $6x^2 + xy - 2y^2 + 7x + 7y - 3 = (2x - y + 3)(3x + 2y - 1)$   
㉜  $12 - 25x + 12x^2 + 4y - 10xy - 8y^2 = (4x + 2y - 3)(3x - 4y - 4)$   
㉝  $bc(b - c) + ca(c - a) + ab(a - b) = (a - b)(a - c)(b - c)$   
㉞  $ab(a + b) + bc(b + c) + ca(c + a) + 2abc = (a + b)(b + c)(c + a)$   
㉟  $(a + b)(b + c)(c + a) + abc = (a + b + c)(ab + bc + ca)$   
㊱  $a^2(b - c) + b^2(c - a) + c^2(a - b) = (a - b)(a - c)(b - c)$   
㊲  $a^3(b - c) + b^3(c - a) + c^3(a - b) = (a - b)(a - c)(b - c)(a + b + c)$   
㊳  $a^4(b - c) + b^4(c - a) + c^4(a - b) = (a - b)(a - c)(b - c)(a^2 + ab + ac + b^2 + bc + c^2)$   
㊴  $x^4 - 13x^2 + 36 = (x - 2)(x + 2)(x - 3)(x + 3)$   
㊵  $x^4 - 1 = (x - 1)(x + 1)(x^2 + 1)$   
㊶  $(x^2 - x - 3)(x^2 - x - 5) - 3 = (x + 1)(x - 2)(x + 2)(x - 3)$   
㊷  $(x^2 + 6x + 3)(x^2 + 6x + 7) + 4 = (x + 1)^2(x + 5)^2$   
㊸  $(x - 1)(x - 2)(x + 3)(x + 4) - 24 = x(x + 2)(x^2 + 2x - 11)$   
㊹  $x^4 + x^2 + 1 = (x^2 - x + 1)(x^2 + x + 1)$

$$\textcircled{45} \quad x^4 - 22x^2 + 9 = (x^2 - 4x - 3)(x^2 + 4x - 3)$$

$$\textcircled{46} \quad 4x^4 + y^4 = (2x^2 - 2xy + y^2)(2x^2 + 2xy + y^2)$$

$$\textcircled{47} \quad x^6 - 1 = (x - 1)(x^2 + x + 1)(x + 1)(x^2 - x + 1)$$

$$\textcircled{48} \quad abx^2 - (a^2 + b^2)x + ab = (bx - a)(ax - b)$$

$$\textcircled{49} \quad (a + b)^2x^2 - 2(a + b)(m + n)xy + (m + n)^2y^2 = (ax + bx - my - ny)^2$$

$$\textcircled{50} \quad (a + b)^2 - 8(a^2 - b^2) + 16(a - b)^2 = (3a - 5b)^2$$

$$\textcircled{51} \quad (a + b)^2 + 2(a + b)(c + d) + (c + d)^2 = (a + b + c + d)^2$$

$$\textcircled{52} \quad (m - 2)x^2 + (3m - 7)x + 2m - 5 = (x + 1)(mx + 2m - 2x - 5)$$

$$\textcircled{53} \quad (b + c - 2a)x^2 + (c + a - 2b)x + a + b - 2c = -(x - 1)(2ax + a - bx + b - cx - 2c)$$

$$\textcircled{54} \quad (x + y)^6 - (y - x)^6 = 4xy(x^2 + 3y^2)(3x^2 + y^2)$$

$$\textcircled{55} \quad a^4 + b^4 + c^4 - 2b^2c^2 - 2c^2a^2 - 2a^2b^2 = (a - b - c)(a + b - c)(a - b + c)(a + b + c)$$

※因数分解の⑤, お配りした問題が  $(a + b)(b + c)(c + a) - abc$  となってましたが, 正しくは  $(a + b)(b + c)(c + a) + abc$  です。お詫びして訂正いたします。